1. a)

$$a\begin{pmatrix}1\\-2\end{pmatrix} - b\begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}$$

$$\begin{array}{llr} \text{I} & a - b\cdot(-3) = 3 & |\cdot -2 \\ \text{II} & a\cdot(-2) - b\cdot 2 = 2 & \end{array}$$

$$\begin{array}{llr} \text{I} & a\cdot(-2) - b\cdot 6 = -6 & \\ \text{II} & a\cdot(-2) - b\cdot 2 = 2 & \text{I} - \text{II} \end{array}$$

$$-b\cdot 4 = -8$$

$$b = \underline{\underline{2}}$$

$\hookrightarrow$ in I.

$$a - 2\cdot(-3) = 3$$
$$a + 6 = 3$$
$$a = \underline{\underline{-3}}$$

b) $\begin{pmatrix}3\\1\\2\end{pmatrix} = \vec{a}$ $\begin{pmatrix}-1\\0\\4\end{pmatrix} = \vec{b}$ $\begin{pmatrix}0\\5\\-1\end{pmatrix} = \vec{c}$

Winkel:

$$\cos\sphericalangle(\vec{a};\vec{b}) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{-3+8}{\sqrt{14}\cdot\sqrt{17}}$$

$$\hookrightarrow \sphericalangle(\vec{a};\vec{b}) = \arccos\left(\frac{5}{\sqrt{238}}\right)$$

$$\sphericalangle(\vec{a};\vec{b}) = \underline{71{,}1^\circ}$$

$$\sphericalangle(\vec{a},\vec{c}) = \arccos\left(\frac{5-2}{\sqrt{14}\cdot\sqrt{26}}\right)$$

$$\sphericalangle(\vec{a},\vec{c}) = \underline{81{,}0^\circ}$$

$$\sphericalangle(\vec{b},\vec{c}) = \arccos\left(\frac{-4}{\sqrt{17}\cdot\sqrt{26}}\right)$$

$$\sphericalangle(\vec{b},\vec{c}) = \underline{101{,}0^\circ}$$

↳ Der Winkel zwischen $\vec{a}$ und $\vec{b}$ ist am kleinsten.

<u>Kreuzprodukte:</u>

$$\vec{a}\times\vec{b} = \begin{pmatrix} 4-0 \\ -2-12 \\ 0+1 \end{pmatrix} = \begin{pmatrix} 4 \\ -14 \\ 1 \end{pmatrix}$$

$$\vec{a}\times\vec{c} = \begin{pmatrix} -1-10 \\ 0+3 \\ 15-0 \end{pmatrix} = \begin{pmatrix} -11 \\ 3 \\ 15 \end{pmatrix}$$

$$\vec{b}\times\vec{c} = \begin{pmatrix} 0-20 \\ 0+1 \\ -5-0 \end{pmatrix} = \begin{pmatrix} -20 \\ 1 \\ -5 \end{pmatrix}$$

Beträge berechnen um längstes Vektorprodukt heraus zu finden

$$|\vec{a}\times\vec{b}| = \sqrt{16+196+1} = \sqrt{213}$$

$$|\vec{a}\times\vec{b}| = \sqrt{121+9+225} = \sqrt{355}$$

$$|\vec{b}\times\vec{c}| = \sqrt{400+1+25} = \sqrt{426}$$

↳ Das Kreuzprodukt $\vec{b}\times\vec{c}$ ist am längsten

c) $\begin{pmatrix} -5 \\ 2x \\ 3 \\ x \end{pmatrix} = \vec{a}$ $\begin{pmatrix} x \\ x \\ 2 \\ -3 \end{pmatrix} = \vec{b}$

$$\sphericalangle \vec{a}\vec{b} = 90^\circ$$

↳ $\cos \sphericalangle \vec{a}\vec{b} = 0$

$$0 = \frac{-5x + 2x^2 + 6 - 3x}{\sqrt{25+4x^2+9+x^2}\ \sqrt{x^2+x^2+4+9}}$$

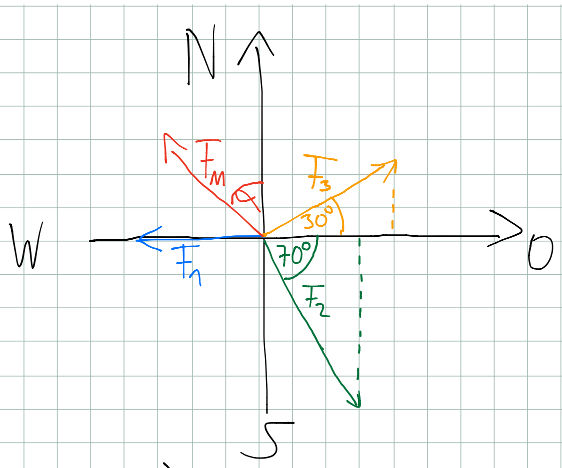
$$0 = 2x^2 - 8x + 6$$

$$0 = x^2 - 4x + 3$$

$$x_{1/2} = 2 \pm \sqrt{4-3}$$

$$x_1 = \underline{\underline{3}} \qquad x_2 = \underline{\underline{1}}$$

2.



$F_1 = 12\,kN$
$F_2 = 23\,kN$
$F_3 = 15\,kN$

$$\vec{F_1} = \begin{pmatrix} -12 \\ 0 \end{pmatrix}$$

$$\vec{F_2} = \begin{pmatrix} \cos(70°) \cdot 23 \\ -\sin(70°) \cdot 23 \end{pmatrix}$$

$$\vec{F_3} = \begin{pmatrix} \cos(30°) \cdot 15 \\ \sin(70°) \cdot 15 \end{pmatrix}$$

$$\vec{F_M} = -\left(\vec{F_1} + \vec{F_2} + \vec{F_3}\right)$$

$$\vec{F_M} = -\left(\begin{pmatrix} -12 \\ 0 \end{pmatrix} + \begin{pmatrix} \cos(70°) \cdot 23 \\ -\sin(70°) \cdot 23 \end{pmatrix} + \begin{pmatrix} \cos(30°) \cdot 15 \\ \sin(30°) \cdot 15 \end{pmatrix}\right)$$

$$\vec{F_M} = -\begin{pmatrix} 8{,}857 \\ -14{,}113 \end{pmatrix}$$

$$\cos\sphericalangle(\vec{F_3}; \vec{F_M}) = \frac{8{,}857 \cdot \cos(30°) \cdot 15 - 14{,}113 \cdot \sin(30°) \cdot 15}{\sqrt{(8{,}857)^2 + (-14{,}113)^2} \cdot \sqrt{(\cos(30°) \cdot 15)^2 + (\sin(30°) \cdot 15)^2}}$$

$$\sphericalangle(\vec{F_3}; \vec{F_M}) = \arccos(0{,}0363)$$

$\sphericalangle(\vec{F_3}, \vec{F_M}) = 87{,}92°$

$\hookrightarrow \alpha = 87{,}92° + 30°$

$\alpha = \underline{\underline{117{,}92°}}$

3.)

$m_A = 0{,}15\,kg \qquad m_B = 0{,}15\,kg$

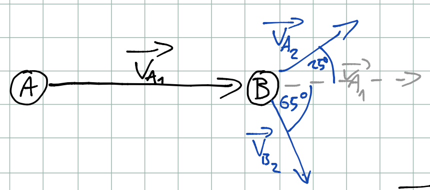
$v_A = 4{,}2\,m\,s^{-1}$

a) Snooker, da die Kugeln beim Pool 0,17kg wiegen.

b)

vor dem Stoß

nach dem Stoß



$$\vec{p}_{A_1} = \vec{p}_{A_2} + \vec{p}_{B_2}$$

$$m \cdot \vec{v}_{A_1} = m \cdot \vec{v}_{A_2} + m \cdot \vec{v}_{B_2}$$

$$\vec{v}_{A_1} = \vec{v}_{A_2} + \vec{v}_{B_2}$$

c)

c) $\sphericalangle(\vec{V_{A_1}}; \vec{V_{A_2}}) = 25°$ $\sphericalangle(\vec{V_{A_1}}; \vec{V_{B_2}}) = 65°$ $\sphericalangle(\vec{V_{A_2}}; \vec{V_{B_2}}) = 90°$

$$\vec{V_{A_1}} = \begin{pmatrix} 4{,}2 \\ 0 \end{pmatrix} \quad \vec{V_{A_2}} = \begin{pmatrix} x_A \\ y_A \end{pmatrix} \quad \vec{V_{B_2}} = \begin{pmatrix} x_B \\ y_B \end{pmatrix}$$

$$\begin{pmatrix} x_A \\ y_A \end{pmatrix} + \begin{pmatrix} x_B \\ y_B \end{pmatrix} = \begin{matrix} 4{,}2 \\ 0 \end{matrix}$$

$$\cos \sphericalangle(\vec{V_{A_1}}; \vec{V_{A_2}}) = \frac{\cancel{4{,}2} \cdot x_A}{\cancel{4{,}2} \cdot \sqrt{x_A^2 + y_A^2}}$$

$$\cos \sphericalangle(\vec{V_{A_1}}; \vec{V_{B_2}}) = \frac{\cancel{4{,}2} \cdot x_B}{\cancel{4{,}2} \cdot \sqrt{x_B^2 + y_B^2}}$$

$$\cos \sphericalangle(\vec{V_{A_2}}; \vec{V_{B_2}}) = 0 = \frac{x_A \cdot x_B + y_A \cdot y_B}{\sqrt{x_A^2 + y_A^2} \cdot \sqrt{x_B^2 + y_B^2}}$$